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雷暴活动所引起的地球大气电场分布问题

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PROBLEMS ON THE GEOGRAPHICAL DISTRIBUTION OF THE ATMOSPHERIC ELECTRIC FIELD INDUCED BY THUNDERSTORM ACTIVITIES*

by

Lee Chi-chen and Lin Chi-zhang

Atmospheric electricity has been a formal subject of study for more than two hundred years. The wealth of observational evidence accumulated during the long period in the past reveals that the electrical phenomena observed on fine days are distinctly different from those associated with bad weather. The field of electric potential in regions of fine weather varies uniformly with the activities of thunderstorms on a global scale. Electric phenomena in the atmosphere over good and bad weather regions are thus interrelated [1].

The world-wide nature of atmospheric electricity is generally well known. However, a satisfactory theoretical explanation on the cause of electric activities in the atmosphere is still lacking.

Two relatively important theories have been proposed, namely, Wilson's theory of spherical condenser and the theory of Frenkel. Wilson [2] has considered that the earth and the ionosphere form two poles of a spherical condenser and that thunderstorms are equivalent to a dynamo. Due to over simplification of the problem, his treatment is unable to account for the observed fact that the distribution of atmospheric charges occur in the troposphere instead of in the ionosphere. Moreover, his model is too crude for a rigorous analysis to be made by means of mathematical

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physics. In Frenkel's theory [1, 3], a zero potential at the earth's surface is assumed or the total electric charge or the entire surface of the earth is taken to be zero. Consequently, his solution to the problem of an infinitely large plane conductor earthed to zero electric potential, or of a neutral conductor deviates markedly from actual atmospheric conditions and the results cannot be regarded as conclusive.

In ~~this~~ the paper, the electromagnetic equation in spherical coordinates is first derived with due consideration on the actual electrical conductivity of the atmosphere and the correct selection of conditions for solution. The distribution of the atmospheric electric field induced by thunderstorm activities is then evaluated. The results are found to be better than those given by Wilson and Frenkel.

I. THE BASIC EQUATION AND CONDITIONS FOR SOLUTION

For the analysis of the electric processes in the atmosphere the Maxwell electromagnetic equations may be expressed in conventional symbols as:

$$\left.\begin{aligned} &\nabla \cdot \mathbf{E} = 4\pi\rho, \\ &\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \\ &\nabla \cdot \mathbf{B} = 0, \\ &\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}. \end{aligned}\right\} \tag{1}$$

The conservation of electric quantity requires that

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{j}. \tag{2}$$

In addition to the conduction current $\mathbf{j}_c$ which obeys Ohm's law, a current $\mathbf{j}'$ may also be induced by an external electromotive potential

in the atmosphere. The generation and separation of electric charge in cloud give rise to electric currents which do not obey Ohm's law. Therefore, we have

$$\mathbf{j} = \mathbf{j}_c + \mathbf{j}', \tag{3}$$

$$\mathbf{j}_c = \lambda \mathbf{E}, \tag{4}$$

where λ denotes the atmospheric conductivity.

In the application of the above equations to the analysis of electric processes in the atmosphere, it is more convenient to use a spherical polar coordinate system (r, θ, φ) with the origin at the center of the earth. Calculations are made on the following three basic assumptions:

(a) Quasi-stability: As the relaxation time of the electrical processes in the earth-atmosphere system is relatively small [4], it is considered that a quasi-equilibrium state may exist at any particular time; thus we have $\partial/\partial t \doteq 0$.

(b) According to observations, the electrical conductivity varies exponentially with height [2, 5] and is given by

$$\lambda(r, \theta, \varphi) = \lambda_0(\theta, \varphi)e^{a(\theta, \varphi)(r-r_0)}, \tag{5}$$

where r_0 denotes the radius of the earth.

(c) Horizontal quasi-uniformity: Electrical conductivity in the atmosphere is essentially a function of height. Although its value may vary from place to place, the horizontal gradient is much smaller than

the vertical gradient. Thus terms with the horizontal gradient of λ in the equation may be neglected.

The first assumption gives $\partial \mathbf{B}/\partial t = 0$ and $\nabla \times \mathbf{E} = 0$. Hence the electric potential V may be introduced as $\mathbf{E} = -\nabla V$. Since $\partial\rho/\partial t = 0$, we have

$$\nabla \cdot \mathbf{j} = -\lambda\nabla^2 V - \nabla\lambda \cdot \nabla V + \nabla \cdot \mathbf{j}' = 0. \tag{6}$$

The generation and separation of electric charge in cloud form one type of "external" electromotive potential and give rise to an electric current $\mathbf{j}'$, which maintains several centers of electric charge in cloud. A positive (negative) charge center Q_i at $(r_i, \theta_i, \varphi_i)$ acts as a sink (source) of $\mathbf{j}'$ but as a source (sink) of $\mathbf{j}_c$. This leads to

$$-\nabla \cdot \mathbf{j}' = \nabla \cdot \mathbf{j}_c = -4\pi Q_i\lambda_i\delta(r - r_i)\delta(\theta - \theta_i)\delta(\varphi - \varphi_i), \tag{7}$$

where Q_i and λ_i denote the quantity of electric charge and the electric conductivity at a particular charge center respectively, while $\delta(x)$ represents a unit pulse function. At this stage, we may treat each charge center as a point charge and then extend the treatment to a more general distribution of electric charge.

By substituting Equation (7) into (6) and considering the second and third assumptions, we obtain

$$\nabla^2 V + \alpha\frac{\partial V}{\partial r} = -\sum_i 4\pi Q_i\delta(r - r_i)\delta(\theta - \theta_i)\delta(\varphi - \varphi_i), \tag{8}$$

where $\sum_i$ represents a summation of all electric charge at the individual charge centers i in the atmosphere.

Since the earth is a good conductor, the earth's surface $r = r_0$ may be taken as an equipotential surface. If we assume as usual that the electric potential is zero at infinity, then the potential at the surface of the earth V_0 remains to be determined. Frenkel [3] has taken V_0 as zero and this is one of the factors which make his approach unsuccessful. According to the first assumption, the time-variation of the total electric charge Q_{earth} at the earth's surface may be expressed as $\partial Q_{earth}/\partial t \doteq 0$. Thus the sum of all electric currents at the earth's surface S_0 should be equal to zero.

The above formulation represents an attempt of finding a solution to the following boundary-value problem:

$$\nabla^2 V + \alpha \frac{\partial V}{\partial r} = -\sum 4\pi Q_i \delta(r - r_i)\delta(\varphi - \varphi_i)\delta(\theta - \theta_i), \tag{8}$$

$$V(r_0, \theta, \varphi) = V_0, \quad \lim_{r \to \infty} V(r, \theta, \varphi) = 0, \tag{9}$$

$$\iint_{S_0} \left(\lambda \frac{\partial V}{\partial r} + j' \right)\Big|_{r=r_0} dS = 0. \tag{10}$$

II. SOLUTION OF EQUATION AND CONCLUSION

By making use of the second assumption and the fact that $\alpha/r \ll \alpha^2/4$, we may, after a change of variables $V(r, \theta, \varphi) = u(r, \theta, \varphi) e^{-\frac{\alpha(\theta, \varphi)}{2}(r - r_0)}$, transform Equation (8) into a typical elliptic partial differential equation which gives a solution to the above boundary-value problem [5] in units of coulomb-volt-km as:

$$V(r, \theta, \varphi) \doteq 3 \times 10^{5} \sum_i Q_i e^{-\frac{1}{2}\alpha(\theta, \varphi)(r - r_i)} \left[\frac{e^{-\frac{1}{2}\alpha(\theta, \varphi) R_i}}{R_i} - \frac{r_0}{r_i} \frac{e^{-\frac{1}{2}\alpha(\theta, \varphi) R_i'}}{R_i'} \right] + V_0 e^{-\alpha(\theta, \varphi)(r - r_0)}, \tag{11}$$

where

$$r_i' = \frac{r_0^2}{r_i},\quad R_i = \sqrt{r^2 + r_i^2 - 2rr_i\cos\gamma},\quad R_i' = \sqrt{r^2 + r_i'^2 - 2rr_i'\cos\gamma}$$

$$\cos\gamma = \cos\theta\cos\theta_i + \sin\theta\sin\theta_i\cdot\cos(\varphi - \varphi_i)$$

while R_i and R_i' denote the distance from $(r_i,\ \theta_i,\ \varphi_i)$ and $(r_i',\ \theta_i,\ \varphi_i)$ to the point of observation $(r,\ \theta,\ \varphi)$ respectively.

Direct verification reveals that Equation (11) satisfies Equation (8). The boundary condition of Equation (9) is satisfied when $r_i'/r_0 \doteq 1$, because the first term in the above expression is approximately zero at $r = r_0$ and the second term in Equation (11) becomes V_0 at $r = r_0$.

The first term on the right-hand side of Equation (11) represents the electric field induced by the charge-centers in cloud, while the second term denotes the charge distribution of the conducting sphere (the earth). Thus they may be denoted by $V_1(r,\ \theta,\ \varphi)$ and $V_2(r,\ \theta,\ \varphi)$ respectively. In order to determine the coefficient V_0 in $V_2(r,\ \theta,\ \varphi)$, we may substitute Equation (11) into Equations (10) to get

$$V_0 = -\frac{\oint\!\!\oint\left[\lambda(r_0,\theta,\varphi)\frac{\partial V_1}{\partial r}\Big|_{r=r_0} + j'\Big|_{r+r_0}\right]dS}{\oint\!\!\oint_{S_0}\lambda(r_0,\theta,\varphi)\alpha(\theta,\varphi)\,dS} = -I_tR_t, \qquad (12)$$

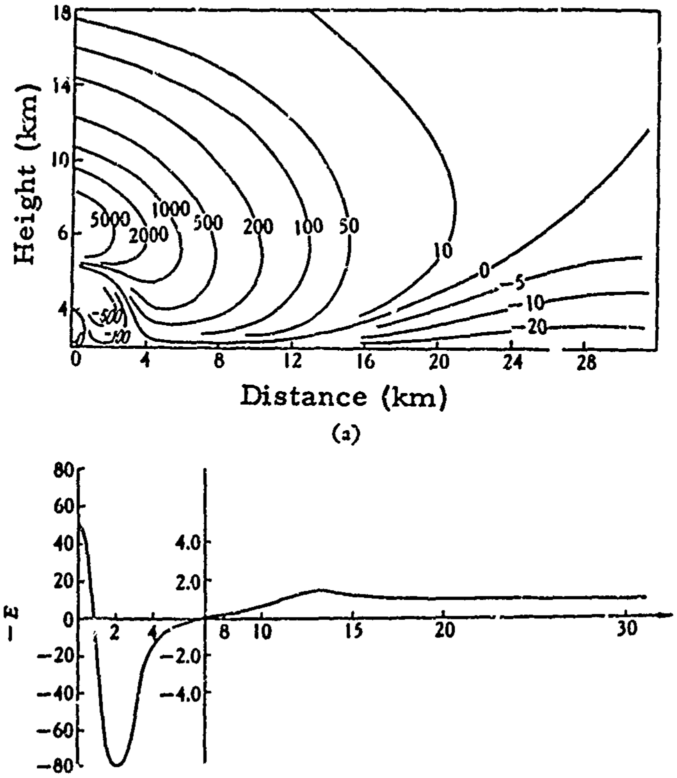
where R_t denotes the total resistance of the earth-atmosphere system and I_t the sum of the conduction currents of all bad weather regions over the world, currents induced by lightning and those released by sharp points and precipitation. According to References [2] and [6], I_t is about 1530-1800 amperes and R_t about 200 ohms, giving V_0 equal to -3.06 to -3.6×10^5 volts.

The above results lead to the following points for discussion:

(a) Differentiation of $V(r, \theta, \varphi)$ with respect to r gives the potential gradient at the surface of the earth as:

$$\left.\frac{\partial V(r,\theta,\varphi)}{\partial r}\right|_{r=r_0} = 2\sum_i \theta_i (r_i - r_0)\cos\gamma \left[\frac{e^{\frac{\alpha}{2}(r_i-r_0)-\frac{\alpha}{2}R_i}}{R_i^2}\left(\frac{\alpha}{2}+\frac{1}{R}\right)\right] - \alpha(\theta,\varphi)V_0, \quad (13)$$

The results of the computed values of $V(r, \theta, \varphi)$ and $\partial V/\partial r\big|_{r=r_0}$ are shown in Figure 1.



(b)

Figure 1

(a) Distribution of electric potential in the region of a thunderstorm (Unit: 10^4 volts.)

(b) Distribution of the surface electric field in the region of a thunderstorm (Unit: volts/cm.)

(b) The first terms in Equations (11) and (13) represent the field of electric charge in cloud. It is noted that one additional factor $\exp\left[+\frac{a}{2}(r_i - r_0) - \frac{a}{2} R_i\right]$ is involved in comparison with similar terms for a static environment problem. Thus it is shown that the effect of a point source decreases with distance more rapidly in an atmosphere with non-uniform conductivity than in a non-conducting atmosphere. This point is worthy of attention because its negligence may lead to an underestimation of the values of Q_i in assessing the potential gradient at the earth's surface in the vicinity of thunderstorms.

(c) The distribution of electric potential in fine weather regions at a distance of more than 10 km from a point source may be determined by the second term $V_2(r, \theta, \varphi)$ in Equations (9) and (11), which represents the field of negative charge of the earth. It may be noted that $V_2 \to 0$ when $r \to \infty$ and $V \to V_0 < 0$ when $r \to 0$, indicating that equipotential conditions exist at the earth's surface and at the upper levels. The potential difference is given by $\Delta V = -V_0 = I_t R_t$, i.e., the upper-air electric potential is higher than that at the earth's surface by 300 to 400 kilovolts. At the earth's surface, we have $\partial V_2/\partial r = -aV_0 \sim 108$ volts/m. These values are found to be in a good agreement with observations.

(d) The stronger the global thunderstorm activity the larger is the value of I_t, giving a higher value of $-V_0$ correspondingly. This gives rise to the so-called "uniform variation" in the electric field. Since the value of $a(\theta, \varphi)$ may vary with space and time, changes in the potential gradient at the earth's surface, $\partial V_2/\partial r\big|_{r=r_0} = a(\theta, \varphi)V_0$ are only uniform over the sea where a is approximately constant.

(e) In fine weather regions, we have $V = V_0 e^{-\alpha(\theta,\varphi)(r-r_0)}$ for the electric potential, $\frac{\partial V}{\partial r} = -\alpha V_0 e^{-\alpha(r-r_0)} > 0$ for the potential gradient, $\rho = -\frac{1}{4\pi}\frac{\partial^2 V}{\partial r^2} = -\frac{\alpha^2 V_0}{4\pi} e^{-\alpha(r-r_0)} > 0$ for the charge density and $Q_{earth} = -\frac{1}{4\pi}\oiint_{S_0} \frac{\partial V_2}{\partial r}\Big|_{r=r_0} dS < 0$ for the total charge at the earth's surface. Thus, the electric potential increases exponentially with height, while the potential gradient and the charge density decrease exponentially with height. These are in good agreement with average observed conditions. In addition, the results of computations show that the distribution of positive charge occurs in the troposphere with negative charge near the earth's surface.

In the above calculations, the charge-centers in cloud are treated as point sources. The derived results may easily be generalized to cover a continuous distribution of electric charge in the form of

$$\begin{aligned} V(r,\theta,\varphi) &= V_0 e^{-\alpha(\theta,\varphi)(r-r_0)} + \\ &+ \sum_i \left\{ \iiint_{\tau_i} \frac{1}{|\mathbf{r}_i - \mathbf{r}|} \rho(\mathbf{r}_i) e^{\frac{\alpha}{2}[(r_i-r)-|\mathbf{r}_i-\mathbf{r}|]} d\mathbf{r}_i - \right. \\ &\left. - \iiint_{\tau_i} \frac{1}{|\mathbf{r}'_i - \mathbf{r}|} \frac{r_0}{r_i} \rho(\mathbf{r}_i) e^{\frac{\alpha}{2}[(r_i-r)-|\mathbf{r}'_i-\mathbf{r}|]} d\mathbf{r}_i \right\}, \end{aligned} \tag{14}$$

where $\mathbf{r}$ denotes the position vector, while $\rho(\mathbf{r}_i)$ and τ_i represent the charge density distribution of the i^{th} charge-center and the total volume respectively. In discussing the electric field in the vicinity of a charge-center, it is necessary to determine accurately the distribution of charge density $\rho(\mathbf{r}_i)$ [7]. However, the requirement does not affect the foregoing conclusive remarks.

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